Wage and Employment Determination in a Dynamic Insider-Outsider Model*

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Abstract

In this paper, I develop a differential insider-outsider game in which a union of corporative incumbents chooses the wage of its members by taking into account the optimal employment policy of a firm that, in turn, is assumed to decide the number of outsiders to hire in a spot labour market. Under the assumption that incumbents cannot be fired and commit themselves to a given path of wages, I demonstrate that such a game displays an open-loop Stackelberg equilibrium in which the initial stock of insiders pins down the trajectories of incumbents, entrants and insider wages. Moreover, resorting to numerical simulations, I show that adjustments towards the steady-state equilibrium occur through asymmetric oscillations that mimic the decline of union membership and union wage premia observed in the US all over the last twenty years. In addition, I show that the model provides a positive relationship between the labour market power of the insider union and the impatience of the firm.

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1 Introduction

The insider-outsider theory of employment and unemployment rests on the assumption that there is a fundamental asymmetry in the wage setting process between incumbent workers (insiders) and unemployed workers who are looking for a job (outsiders). On the one side, relying on labour turnover costs and/or firm-specific skills that may create a productivity premium, insiders are assumed to be endowed with a strong bargaining power in the wage setting process – sometimes strengthened via formal or informal unions – and to exploit this power in order to maximize their pay and foster their employment opportunities. On the other side, outsiders are assumed to have no market power and when they have the chance to find a job, unless they become insiders, their wage is usually quite close to the reservation level and they have no say over their employment prospects.

Undoubtedly, the main backers of the insider-outsider theory are Lindbeck and Snower (1984, 1989, 2002) who developed a long array of works that uncover a wide range of consequences in terms of wages and (un)employment triggered by the interest conflict established between insiders and outsiders in the labour market. Relying on that conflict, the two authors provide a sound rationale for a number of puzzling stylized facts that characterized the macroeconomic experience of Western economies during the 80s and the 90s such as the existence and persistence of involuntary unemployment, the inflexibility of insiders' wages, trade union corporatism and the asymmetry of wage-employment movements observed during recessions and expansions.[1]

In the latest survey article dedicated to the theoretical approach that they contributed to start, Lindbeck and Snower (2002) point out that one of the most complex and still open question of the insider-outsider theory is the way in which employment and wages move through time in response to labour market shocks. Specifically, while there are several insider-outsider contributions that show how to fix the levels of employment and wages at any point in time, dynamic insider-outsider models – despite some fair exceptions – lag somehow behind. Similarly, in another survey, Sanfey (1995) argues that modifying traditional union models to consider the distinction between insiders and outsiders is straightforward, but that task is actually much more complicated in dynamic models.

In confirmation of the arguments recalled above, the dynamic insider-outsider literature counts a limited number of contributions. For instance, Solow (1985) outlines a two-period insider-outsider model but he is mainly concerned about what happens in the first and he does not derive any explicit dynamic law for employment and wages.[2] Drazen and Gottfries (1990)

[1] In the same years, other prominent authors contributed to that theory. For example, Carruth and Oswald (1987) provided a model of union behaviour grounded on the distinction between insider and outsider workers and discussed some related macroeconomic implications such as the possibility that productivity improvements can feed into pure wage increases for insiders with no, or minor, effects on employment. Furthermore, Blanchard and Summers (1986) as well as Gottfries and Horn (1987) relied on insider-outsider relations to explain the strong persistence of unemployment observed in European countries during the 80's.

[2] Recognizing the inherent dynamic of the insider-outsider hypothesis, a similar exercise is carried out by Vetter and Andersen (1994).

set forth a dynamic optimizing insider-outsider framework developed over an infinite horizon, but they model the evolution of wages and employment by means of few discrete realizations of the two variables. Huizinga and Schiantarelli (1992) develop a discrete-time model in which a firm and an insider union efficiently bargain over the wage and employment but the two authors focus only on the path of employment adjustments driven by productivity shocks. Fukuda and Owen (2008) build a series of overlapping-generations (OLG) models with human capital accumulation in which the insider-outsider dichotomy is conveyed in terms of firm-specific versus general skills.[3] More recently, Galì (2016) sets forth a New Keynesian DSGE model with an insider-outsider labour market by exploring the implications of that environment for the design of optimal monetary policies when unemployment is strongly persistent.

To the best of my knowledge, the present contribution is the first attempt to provide a dynamic insider-outsider model developed within an optimal control framework with continuous time and infinite horizon. Specifically, I build a parsimonious differential game in which a corporative union of insider workers whose members cannot be fired is called in to choose the common wage trajectory of incumbents by taking into account the optimal employment path selected by a representative profit-maximizing firm that, in turn, is assumed to decide the number of outsiders – or entrants – to hire in each instant on a spot labour market. In detail, in my game new hirings boost union membership at a fixed rate and newly hired workers are assumed to be costly for recruiting entrepreneurs. Furthermore, the reservation wage of outsiders as well as the redundancy rate of insiders are taken as exogenously given.

Within that strategic framework, the dynamic interaction between the decisions of the firm and the ones of the union describes how wages and employment are continuously determined and move through time in a typical insider-outsider economy. In my proposal, the labour input of incumbents follows a stock-adjustment process and its marginal cost – the insider wage – evolves according to the preferences of unionized workers who are in the position to manipulate the recruiting decisions of the entrepreneur without caring about the welfare of unemployed people.

The theoretical insider-outsider setting developed in this paper differs from recent dynamic union models with bargaining in many directions. For instance, consistently with the search-and-matching framework with a monopoly union set forth by Krusell and Rudanko (2016), the union of insiders described in my differential game embodies in its optimization problem an intertemporal constraint that conveys the optimal hiring decisions of the representative firm. From a dynamic perspective, this feature is at odds with respect DSGE models with wage bargaining such as Mattesini and Rossi (2008, 2009) and Zanetti (2007) in which unions act as static Stackelberg leaders that do not consider the effects of their optimal wage trajectory on the dynamic path of hirings decided by employers.

Other distinguishing marks of my dynamic insider-outsider model are the definition of union membership as well as the treatment of labour inputs. On the one hand, as opposed to Morin

[3]Different OLG models in which the insider-outsider distinction is concerned are given by McCausland (1998) and Begg (1988).

(2017), Krusell and Rudanko (2016), Alvarez and Shimer (2014), Zanetti (2007) and Delacroix (2006), union members do not include unemployed workers. Obviously, at the firm level, this makes a sharp distinction between internal and external workers.[4] On the other hand, the labour input employed by the firm is split in two distinct components, i.e. the insider and the entrant labour force that are not perfectly substitutable for employers. The former follows a stock-adjustment principle whose inflows are not described by a matching function as in Morin (2017), Krusell and Rudanko (2016) and Trigari (2006), but are given instead by the fixed share of entrants that join the insider union. The latter is a flow variable that represents newly hired workers who suffer a productivity gap with respect to incumbents. Such a particular characterization deserves to be stressed since Alvarez and Shimer (2014), Mattesini and Rossi (2008, 2009) and Zanetti (2006, 2007) treat labour input as a single flow variable under the control of employers. In addition, the insider union described in this paper is assumed to set a smooth wage trajectory without relying on any exogenous minimum wage – or wage norm – as is done instead in Alvarez and Shimer (2014), Mattesini and Rossi (2008, 2009) and Delacroix (2006).

The results of my theoretical exploration can be summarized as follows. First, under the assumption that the union of insiders commits itself to a given path of wages and behaves in an egalitarian manner, I demonstrate that the dynamic insider-outsider model economy displays an open-loop Stackelberg equilibrium in which the initial stock of insiders pins down the trajectories of incumbents, entrants and insider wages in a well-determined manner (cf. Dockner et al. 2000). Moreover, resorting to numerical simulations, I show that in the dynamic model under investigation adjustments towards the unique steady-state equilibrium occur through damped asymmetric oscillations that mirrors the parallel decline of union membership and union wage premia observed in the US over the last twenty years (cf. Blanchflower and Bryon, 2004). According to the logic underlying the game played by the workers and the firm, this pattern is a direct implication of the optimal behaviour of unionized insiders that extract rents from entrepreneurs subject to firing limitations (cf. Alvarez and Shimer, 2014; Krusell and Rudanko, 2016). In addition, consistently with models of dynamic bargaining, I show that my intertemporal insider-outsider framework delivers a positive equilibrium relationship between the labour market power of the union and the degree of impatience of the firm (cf. Binmore et al. 1986; Osborne and Rubinstein, 1990).

The paper is arranged as follows. Section 2 develops the theoretical model. Section 3 explores its numerical properties. Finally, Section 4 concludes.

[4] In Morin (2017) union members are either employed or unemployed. When contracting the wage, however, the union considers only the surplus of employed workers because their fall back utility is assumed to coincide with the welfare of unemployed individuals. Therefore, similarly to Huizinga and Schiantarelli (1992), the bargaining process proceeds by weighting the surplus of insiders and the one of the firm.

2 The model

I consider a model economy in which time is continuous and the time horizon is infinite. Within this economy, in each instant – say t – a representative firm starts with a pool of $L_I(t)$ insider workers and it must decide how many outsiders have to be hired in a spot labour market by bearing the implied hiring costs.

Winking at the hypothesis of static insider-outsider models in which fully-fledged employees set their wage as high as possible consistent with their continued employment, I assume that insiders cannot be fired by the firm (cf. Lindbeck and Snower, 1987). Thereafter, following Huizinga and Schiantarelli (1992), I make the hypothesis that incumbents may lose their job at the exogenously given rate b that determines the outflows from employment. By contrast, inflows into employment are given by newly hired workers – or entrants – denoted by $L_E(t)$.

According to Booth (1984), individuals usually join a union only when the expected utility from so doing exceeds the utility from not joining. Similarly, Jones and McKenna (1994) argue that employed workers join a union if the marginal benefit of protection is at least as great as union dues. Aiming at placing entrants and outsiders on the same level and skipping the modelling of workers' preferences, in this paper I assume that a fixed fraction m of outsiders becomes insiders just immediately after their appointment while the remaining become unemployed.[5] Consequently, the dynamic evolution of the stock of incumbent workers can be written as

$$\dot{L}_I(t) = mL_E(t) - bL_I(t) \qquad 0 < m < 1,\ 0 < b < 1 \tag{1}$$

The expression in (1) implicitly defines the way in which outsiders turn into insiders and reveals that the hiring decision of the firm have a direct influence on the evolution of the stock of seasoned workers. In that differential equation, the existence of a positive job-destruction rate for incumbents is a prerequisite for the derivation of a stationary solution in which both $L_I(t)$ and $L_E(t)$ are positive and constant over time. Moreover, eq. (1) has the convenient property that in steady-state the proportion of insiders on the whole labour force employed by the firm is fixed at $m/(m+b) < 1$. Interestingly, when $m = b$, i.e. when the fraction of outsiders that become insiders in each instant is equal to the internal job destruction rate, the steady-state equilibrium is characterized by an equal number of entrants and insiders.

With regard to the production technology, similarly to Guerrazzi (2011), I assume that produced output is a quadratic combination of the employed labour force. Therefore, under the hypothesis that the flow of entrants is less productive than the stock of insiders, the production function of the representative firm is given by

$$Y(t) = A\left(L_I(t) + \phi L_E(t)\right) - \frac{\alpha}{2}\left(L_I^2(t) + L_E^2(t)\right) \qquad A > 0,\ 0 < \phi < 1,\ \alpha > 0 \tag{2}$$

[5] An alternative hypothesis could be to assume that workers who do not become unionized remain employed without the benefits of union membership. The exploration of that assumption under which outsiders become a distinct state variable in addition to the stock of insiders is left to future developments.

where $Y(t)$ is the level of output, A is a measure of the total factor productivity (TFP), ϕ measures the productivity differential between insiders and outsiders whereas α is a parameter that conveys the slope of the marginal productivity for the two categories of workers.

The quadratic specification in eq. (2) has the virtue to deliver linear marginal productivity schedules both for insiders and outsiders whose distance is proportional to the parameter ϕ. Indeed, straightforward differentiation reveals that $\partial Y(t)/\partial L_I(t) = A - L_I(t)$ and $\partial Y(t)/\partial L_E(t) = \phi A - L_E(t)$. As argued by Lindbeck and Snower (1987), the wedge between the productivity of the two different categories of workers – together with labour turnover costs – provides a measure of the underlying degree of insiderness (cf. Manzini and Snower, 2002). Moreover, eq. (2) reveals that the marginal productivity of insiders is independent of the one of outsiders and vice versa, i.e. $\partial^2 Y(t)/(\partial L_I(t)\,\partial L_E(t)) = \partial^2 Y(t)/(\partial L_E(t)\,\partial L_I(t)) = 0$. Consistently with the hypothesis that the union behaves in a corporative manner with the aim of protecting the positions of its members, such a pattern precludes any form of work collaboration between insiders and outsiders (cf. Lindbeck and Snower, 1988). In addition, from an empirical point of view, the expression in eq. (2) implies that the elastiticities of output with respect the two labour inputs – respectively, $\epsilon_{Y,L_I} \equiv L_I(t)(A - L_I(t))/Y(t)$ and $\epsilon_{Y,L_E} \equiv L_E(t)(\phi A - L_E(t))/Y(t)$ – are not constant and this fact appears more consistent with the most recent attempts to estimate actual production functions (cf. Ackerberg et al. 2015).

Denoting by $w_E > 0$ the exogenously given reservation wage that has to be paid to outsider workers, the picture of the instantaneous marginal productivity of the two kinds of workers implied by the production function in eq. (2) is illustrated in Figure 1.

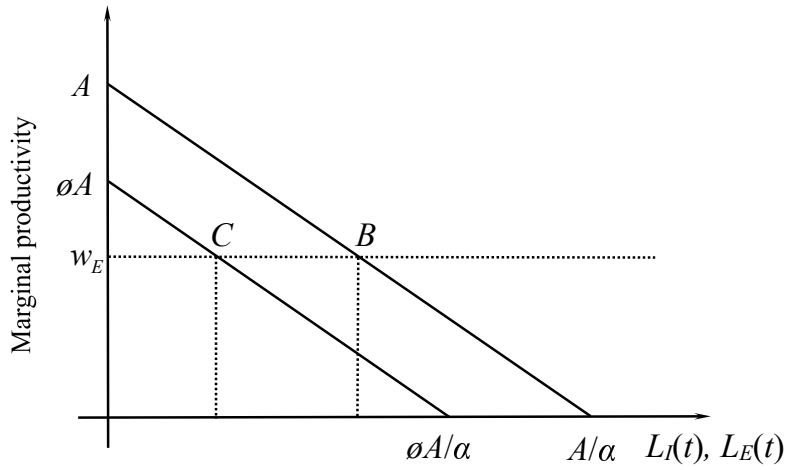

Figure 1: Marginal productivity of insiders and outsiders

The diagram in Figure 1 reveals some interesting features of the marginal productivity of the insiders and the one of the outsiders. Specifically, the former crosses w_E in B and displays a trade-off between the profitability of incumbents and their level of employment. The latter crosses w_E in C and reveals that in the spot labour market there is only one allocation in

which the productivity of entrants coincides with their wage. In this regard, the exogeneity of the reservation wage means that labour supply for new applicants is perfectly elastic so that paying w_E the firm can potentially hire all the outsiders that it wants. Moreover, together with the fixed transformation rate m, that feature qualifies this setting as a partial equilibrium model in which the underlying dynamics is not affected by capital accumulation (cf. Huizinga and Schiantarelli, 1992; Mattesini and Rossi, 2008, 2009). In the remainder of the paper, I will assume that the information conveyed by the two linear schedules depicted in Figure 1 is common knowledge both for workers and the firm. Furthermore, I will consider a situation in which the marginal productivity of outsiders is always above their reservation wage. Formally speaking, as it actually happens in the graph of Figure 1, I will make the hypothesis that $A\phi > w_E$.

As I mentioned in the introduction, insider workers are assumed to be organized in an internal labour union whose task is to set the real wage of its members in an egalitarian manner by taking into account the hiring decisions of the firm.[6] Along the lines of Dertouzos and Pencavel (1981) and Pencavel (1985), I consider the case in which the preferences of the union are given by a log-linear function such as

$$U\left(L_I\left(t\right), w_I\left(t\right)\right) \equiv \beta \log\left(L_I\left(t\right)\right) + \left(1-\beta\right) \log\left(w_I\left(t\right)\right) \qquad 0 < \beta < 1 \tag{3}$$

where β $(1-\beta)$ measures the relative weight of employment (wage) in the union preferences.

The existing literature on wage determination does not provide exhaustive insights on how to model the objective function of a union (cf. Farber, 1986; Gahan, 2002; Kaufman, 2002). As argued by Jones and McKenna (1994), however, it seems reasonable to assume that a union of insiders should care – among other things – about the attained level of membership and the wage earned by its members. Accordingly, eq. (3) implies that the union suffers a loss (experiences a gain) when the stock of incumbent workers decreases (the insider wage increases) whose magnitude does not depend on $w_I\left(t\right)$ $\left(L_I\left(t\right)\right)$. In that expression, a value of β higher (lower) than $1/2$ means that the insider union is employment-oriented (wage-oriented) (cf. Sanfey, 1995; Mattesini and Rossi, 2008, 2009).

The employment component of the union welfare function in eq. (3) deserves some additional remarks. First, the fact the union cares about the level of its membership can be taken as a feature that mirrors its political objectives; indeed, larger organizations are usually assumed to have a stronger ascendancy in the decisional processes in which they are involved (cf. Becker, 1983). In that direction, the union concern about $L_I\left(t\right)$ can provide a rationale for its role of leader played in the differential game under scrutiny. Second, since I'm assuming that insiders cannot be fired, the expression in eq. (3) avoids the kinks and discontinuities in the union welfare that are typical in static insider-outsider models (cf. Carruth and Oswald, 1987). Furthermore, confirming the corporative attitude of incumbents, eq. (3) straightforwardly reveals that the

[6] As argued by Drazen and Gottfries (1990), one may think of a union which organizes an industry and that is large relative to firms in that industry. Delacroix (2006) defines a union with the features assumed in this paper as 'monolithic'.

welfare of outsiders is given no weight in the preferences of unionized workers.

2.1 The problem of the firm

Consistently with Solow (1985), the representative risk-neutral firm is assumed to maximize the discounted flow of its expected profit by taking the trajectory of insider wages as given but – at the same time – considering that its hiring decisions have an impact on the dynamics of incumbent workers, i.e. on the evolution of union's membership. In other words, I assume that the union of insiders is able to commit itself to a given path of wages and then it announces its egalitarian wage policy $\{w_I(t)\}_{t=0}^{\infty}$ at the beginning of the game.[7] Therefore, assuming that hiring costs have a quadratic specification such as the ones suggested by Hamermesh (1995), the problem of the firm can be written as

$$\begin{gathered}\max_{\{L_E\}_{t=0}^{\infty}} \int_{t=0}^{\infty} \exp(-\rho_F t)\left(Y(t) - w_I(t) L_I(t) - L_E(t)\left(w_E + \tfrac{h}{2} L_E(t)\right)\right) dt \qquad \rho_F > 0, \;\; h > 0 \\ \text{s.to} \\ \dot{L}_I(t) = m L_E(t) - b L_I(t) \\ L_I(0) = \overline{L}_I\end{gathered} \tag{4}$$

where ρ_F is the discount rate of the representative firm, h is a measure of the slope of marginal hiring costs, whereas $\overline{L}_I > 0$ is the initial number of insider workers.

Considering the production function in (2) and taking $w_I(t)$ as given, the first-order conditions (FOCs) for the dynamic problem in (4) are the following:

$$A\phi - (\alpha + h) L_E(t) - w_E + m\Lambda(t) = 0 \tag{5}$$

$$\dot{\Lambda}(t) = \Lambda(t)(b + \rho_F) - A + \alpha L_I(t) + w_I(t) \tag{6}$$

$$\lim_{t \to \infty} \exp(-\rho_F t)\, \Lambda(t)\, L_I(t) = 0 \tag{7}$$

where $\Lambda(t)$ is the costate variable on the dynamic constraint describing the evolution of insider workers.

Eq. (5) is the FOC with respect to $L_E(t)$. Moreover, the differential equation in (6) describes the optimal path of $\Lambda(t)$, whereas (7) is the required transversality condition. Regarded as conditions on the time path of $\Lambda(t)$, eq.s (1), (5) and (6) convey the incentive compatibility conditions of a principal-agent problem. Specifically, if the union can suggest a time path for

[7] The hypothesis that the union imposes an identical wage for all the insider workers can be motivated by the broad literature documenting that unions tend to compress the distribution of wages (cf. Alvarez and Shimer, 2014; Krusell and Rudanko, 2016).

$\Lambda(t)$ such that eq.s (1), (5) and (6) are satisfied, then the optimization problem of the firm is solved (cf. Long, 1992).

From an algebraic point of view, the FOCs of the firm problem yields $L_E(t)$ as a function of $\Lambda(t)$. Therefore, if we can find functions $\Lambda(t)$ and $L_I(t)$ that satisfy eq.s. (5) and (6) as well as the boundary condition $L_I(0) = \overline{L}_I$, then the optimal open-loop strategy for the firm is given by

$$L_E(t) = \frac{A\phi - w_E + m\Lambda(t)}{\alpha + h} \tag{8}$$

Eq. (8) reveals that – in each instant – a positive flow of entrants requires that $A\phi + m\Lambda(t) > w_E$. Moreover, according to eq. (8), that flow is stronger, the lower the values of α and h.

2.2 The problem of the union

Continuing the analogy with Solow (1985), the union of insider workers is assumed to set the wage of its members as a Stackelberg leader who takes into account the fact that its membership dynamics is influenced by firm's hiring decisions described by eq.s (6) and (8). Otherwise said, this means that $\Lambda(t)$, i.e. the shadow value of insider employment in the firm's problem, is treated as a state variable in the leader's optimization problem (cf. Dockner et al. 2000). Consequently, the problem of the union is given by

$$\begin{gathered}
\max_{\{w_I\}_{t=0}^{\infty}} \int_{t=0}^{\infty} \exp(-\rho_U t)\left(\beta \log(L_I(t)) + (1-\beta)\log(w_I(t))\right) dt \qquad \rho_U > 0 \\
\text{s.to} \\
\dot{L}_I(t) = m\tfrac{A\phi - w_E + m\Lambda(t)}{\alpha+h} - bL_I(t) \\
\dot{\Lambda}(t) = \Lambda(t)(b+\rho_F) - A + \alpha L_I(t) + w_I(t) \\
L_I(0) = \overline{L}_I
\end{gathered} \tag{9}$$

where ρ_U is the discount rate of the union which is not necessarily equal to the one of the firm.

The FOCs for the problem in (9) are given by

$$\frac{1-\beta}{w_I(t)} + \Psi(t) = 0 \tag{10}$$

$$\dot{\Gamma}(t) = \Gamma(t)(\rho_U + b) - \frac{\beta}{L_I(t)} - \alpha\Psi(t) \tag{11}$$

$$\dot{\Psi}(t) = \Psi(t)(\rho_U - \rho_F - b) - \frac{m^2}{\alpha+h}\Gamma(t) \tag{12}$$

$$\lim_{t\to\infty} \exp(-\rho_U t)\,\Gamma(t)\,L_I(t) = \lim_{t\to\infty} \exp(-\rho_U t)\,\Psi(t)\,\Lambda(t) = 0 \tag{13}$$

where $\Gamma(t)$ and $\Psi(t)$ are, respectively, the costate variable on the dynamic constraint describing the evolution of union membership and the costate variable on $\Lambda(t)$.

Again, eq. (10) is the FOC with respect to $w_I(t)$. Moreover, the differential equations in (11) and (12) define, respectively, the optimal path of $\Gamma(t)$ and $\Psi(t)$, whereas (13) are the required transversality conditions.

Exploiting the result in eq. (10), the solution of the union problem provides the following nonlinear 4×4 system of autonomous differential equations:

$$\begin{pmatrix} \dot{L}_I(t) \\ \dot{\Lambda}(t) \\ \dot{\Gamma}(t) \\ \dot{\Psi}(t) \end{pmatrix} = \begin{pmatrix} \frac{m(A\phi - w_E)}{\alpha + h} \\ -A \\ 0 \\ 0 \end{pmatrix} + \begin{bmatrix} -b & \frac{m^2}{\alpha+h} & 0 & 0 \\ \alpha & b + \rho_F & 0 & -\frac{1-\beta}{\Psi^2(t)} \\ -\frac{\beta}{L_I^2(t)} & 0 & b + \rho_U & -\alpha \\ 0 & 0 & -\frac{m^2}{\alpha+h} & \rho_U - \rho_F - b \end{bmatrix} \begin{pmatrix} L_I(t) \\ \Lambda(t) \\ \Gamma(t) \\ \Psi(t) \end{pmatrix} \tag{14}$$

In addition, the dynamics of $L_E(t)$ and $w_I(t)$ are given by

$$\dot{L}_E(t) = \frac{m}{\alpha + h} \dot{\Lambda}(t) \tag{15}$$

$$\dot{w}_I(t) = \frac{1 - \beta}{(\Psi(t))^2} \dot{\Psi}(t) \tag{16}$$

The differential equations in (15) and (16) show that the dynamics of L_E and w_I over time is proportional to the evolution of, respectively, Λ and Ψ. Moreover, eq. (16) has the intriguing implication that when the union does not care about the wage of its members, i.e., when $\beta \to 1$, the optimal value of w_I tends to be constant over time. Consequently, the employment orientation of the insider union straightforwardly leads to the inflexibility of the insider wage (cf. Lindbeck and Snower, 1984; 1987).

Now, it is worth noting the following important result:

Lemma 1: The value of $\Lambda(0)$ is controllable by the union so that the union itself is able to control the solution of the firm problem.

Proof: See the Appendix.

Whenever $\Lambda(0)$ is controllable by the union, its value cannot be determined independently of the leader's control path of $w_I(t)$. From an economic point of view, this means that in each instant the union of internal workers is able to set an insider wage that leads the firm to hire the number of entrants that is in the best interest of the welfare trajectory of the union itself. In a dynamic perspective, this feature summarizes the essence of the insider-outsider dilemma (cf. Lindbeck and Snower, 1984). In order to achieve that remarkable outcome, the initial value of the costate variable of the union problem associated to $\Lambda(t)$ has to be set to zero (cf. Dockner et al. 2000). Obviously, this means that the FOCs of the union problem must be supplemented by the following transversality condition:

$$\Psi(0) = 0 \tag{17}$$

The transversality condition in (17) has two important implications for the union problem. First, plugging (17) into eq. (10), allows us to state that in the initial stage of the differential

game, and in that stage only, the union – exactly as it would do in a static environment – sets a wage that leads its marginal gain from a wage increase to zero. With log-linear preferences, this means that at the outset of the game the union sets an explosive wage. Intuitively, since it knows that the stock of incumbents is immune from firing, the union tries to set the wage of its members as high as possible to appropriate rents from the firm. This means that at the beginning of the game the union is wage-oriented no matter the shape of its preferences, i.e. no matter the value of β. Thereafter, in the subsequent instants, the union tunes the level of w_I in order to make profitable for the firm to hiring a level of L_E that fulfils its optimal trajectory of membership.

To some extent, the result on controllability echoes the one achieved in the search-and-matching model set forth by Krusell and Rudanko (2016) where – under wage commitment – a positive starting level of incumbents leads the union of workers to hiking up wages by creating a distortion in the optimal hiring decisions of the firm. Moreover, taking into account that the initial stock of incumbents is given, i.e. $L_I(0) = \overline{L}_I$, the transverality condition in eq. (17) implies that the saddle-path determinacy of the 4×4 dynamic system in (14) requires the existence of two negative roots.

2.3 Steady state

Within the model under investigation, steady-state equilibria are defined as the set of quadruplets $\{L_I^*, \Lambda^*, \Gamma^*, \Psi^*\}$ such that $\dot{L}_I(t) = \dot{\Lambda}(t) = \dot{\Gamma}(t) = \dot{\Psi}(t) = 0$. The elements of that set collect allocations in which the insider wage is stable over time and – in each instant – the number of hired outsiders that join the union is equal to the number of retiring insiders, respectively, $w_I^* = -(1-\beta)/\Psi^*$ and $L_E^* = (b/m) L_I^*$.

From a formal point of view, the derivation of $\{L_I^*, \Lambda^*, \Gamma^*, \Psi^*\}$ can be done in four simple steps. First, setting $\dot{\Psi}(t) = 0$ in eq. (12), the value of Ψ^* can be written as a function of Γ^*. Specifically,

$$\Psi^* = \frac{m^2}{M_0}\Gamma^* \tag{18}$$

where $M_0 \equiv (\alpha + h)(\rho_U - (\rho_F + b))$.

Second, setting $\dot{\Gamma}(t) = 0$ in eq. (11) and plugging the result into eq. (18) allows us to write Γ^* as a function of L_I^*, so that

$$\Gamma^* = \frac{\beta M_0}{(M_1 - \alpha m^2) L_I^*} \tag{19}$$

where $M_1 \equiv (\rho_U + b) M_0$.

Third, setting $\dot{\Lambda}(t) = 0$ in eq. (6) and plugging the results into eq.s. (10) and (19), Λ^* can be written as

$$\Lambda^* = \frac{A\beta m^2 + ((1-\beta)(M_1 - \alpha m^2) - \alpha\beta m^2) L_I^*}{\beta (b + \rho_F) m^2} \tag{20}$$

Finally, setting $\dot{L}_I(t) = 0$ in eq. (1) and plugging the results into eq.s (8) and (20) allows us to derive in an unequivocal manner the steady-state level of the insider labour force. Specifically,

$$L_I^* = \frac{\beta m\left(\left(A\phi - w_E\right)\left(\rho_F + b\right) + Am\right)}{b\beta\left(\alpha + h\right)\left(b + \rho_F\right) - \left(1 - \beta\right)\left(M_1 - \alpha m^2\right) + \alpha\beta m^2} \tag{21}$$

Plugging the expression in eq. (21) into eq.s (19) and (20) leads to the determination of the unique steady-state solution of the dynamic system in (14) as well as to pegging the equilibrium pair $\{w_I^*, L_E^*\}$.

Eq.s (18) − (21) are useful to show how the stationary solution of the game is affected by changes in the values of the main parameters' model. Specifically, while the consequences implied by different values of the two discount rates are discussed below by means of some numerical examples, the effects driven by variations of A, ϕ, β, h, and m on the equilibrium triplet $\{L_I^*, L_E^*, w_I^*\}$ are described in the following Propositions:

Proposition 1: Higher (lower) productivity levels, i.e. higher (lower) values of A, of ϕ or both, lead to an increase (decrease) in L_I^*, L_E^* and w_I^*.

Proposition 2: The higher (weaker) the employment orientation of the union, i.e. the higher (lower) the values of β, the higher (lower) the values of L_I^* and L_E^* and the lower (higher) the values of w_I^*.

Proposition 3: An increase (reduction) in marginal hiring costs, i.e. an increase (reduction) in h, lead to a reduction (increase) in L_I^*, L_E^* and w_I^*.

Proposition 4: Higher (lower) values of the transformation rate of outsiders into insiders, i.e. higher (lower) values of m, lead to an increase (reduction) in L_I^* but reduce (increase) L_E^* and w_I^*.

While the results in Propositions 1 and 2 are trivial, the ones in Propositions 3 and 4 require some explanation.[8] In the first place, the effects driven by changes in h described in Proposition 3 are at odds with respect to what happens in a static insider-outsider model where – consistently with a short-run perspective – higher (lower) turnover costs usually allow incumbents to negotiate higher (lower) wage payments (e.g. Lindbeck and Snower, 1987; Coe and Snower, 1997). The rationale for this opposite finding is that, everything else being equal, higher (lower) hiring costs discourage (encourage) the firm from recruiting (to hire) new entrants. In the long-run, this in turn will reduce (increase) the stock of incumbent workers as well as the output produced by the firm. Obviously, the lower (higher) the output of the firm, the lower (higher) the insider wage.

Moreover, the explanation of the results in Proposition 4 is quite similar to the one underlying Proposition 3. Specifically, the higher (lower) the number of entrants who become insiders in each instant, the lower (higher) the incentive for the firm to hire new employees; indeed,

[8]The only thing to notice about Proposition 2 is that when union do not care about its membership, i.e. when $\beta \to 0$, the stationary solution for the insider wage cannot be determined.

once unionized, entrants will have to be paid more than their reservation wage.[9] In addition, the larger (lower) the stock of incumbents, the lower (higher) its marginal productivity and the lower (higher) the insider wage.

2.4 Local dynamics

Given the quadruplet $\{L_I^*, \Lambda^*, \Gamma^*, \Psi^*\}$ derived above, the local dynamics of the model economy in the neighbourhood of the stationary solution is described by the following linear 4×4 system:

$$\begin{pmatrix} \dot{L}_I(t) \\ \dot{\Lambda}(t) \\ \dot{\Gamma}(t) \\ \dot{\Psi}(t) \end{pmatrix} = \begin{bmatrix} -b & \frac{m^2}{\alpha+h} & 0 & 0 \\ \alpha & \rho_F + b & 0 & \frac{1-\beta}{(\Psi^*)^2} \\ \frac{\beta}{(L_I^*)^2} & 0 & \rho_U + b & -\alpha \\ 0 & 0 & -\frac{m^2}{\alpha+h} & \rho_U - \rho_F - b \end{bmatrix} \begin{pmatrix} L_I(t) - L_I^* \\ \Lambda(t) - \Lambda^* \\ \Gamma(t) - \Gamma^* \\ \Psi(t) - \Psi^* \end{pmatrix} \tag{22}$$

Denoting with λ_i, with $i = \{1, 2, 3, 4\}$, the eigenvalues of the Jacobian matrix in (22), the corresponding characteristic equation of the linearized dynamic system can be written as

$$\lambda^4 - 2\rho_U \lambda^3 + \frac{(\rho_U(\rho_F + \rho_U) - \rho_F^2 - 2b(b + \rho_F))(\alpha + h) - 2\alpha m^2}{\alpha + h}\lambda^2 + \Delta_1 \lambda + \Delta_2 + \Delta_3 \tag{23}$$

where the constants Δ_j, with $j = \{1, 2, 3\}$, are given by the following array:

$$\begin{pmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \end{pmatrix} \equiv \begin{pmatrix} \frac{\left(b(b+\rho_F)F + \alpha m^2\right)G - \left(\left(b^2 - \rho_U\rho_F\right)F + \alpha m^2\right)(b + \rho_F - \rho_U) + \alpha\rho m^2}{F} \\ \frac{G\left(\left(b(b+\rho_F)F + \alpha m^2\right)(b + \rho_F - \rho_U) + \alpha m^2(G + \rho_F)\right)}{F} \\ \frac{m^2\left(\alpha(\Psi^*)^2(L^*)^2\left(\left(b^2 - \rho_U\rho_F\right)F + \alpha m^2\right) - \alpha FG^2 + (1-\beta)\beta m^2\right)}{(\Psi^*)^2(L^*)^2F^2} \end{pmatrix} \tag{24}$$

where $F \equiv \alpha + h$ and $G \equiv b + \rho_U$.

Considering that the trace of the Jacobian matrix in (22) is equal to $2\rho_U > 0$ – and implementing the analytical and numerical results derived by Kempt et al. (1980), Dockner (1985) and Dockner and Feichtinger (1991) to the expressions in (23) and (24) – it becomes possible to state that the steady-state allocation $\{L_I^*, \Lambda^*, \Gamma^*, \Psi^*\}$ is a saddle-point characterized by two negative complex conjugate roots, say λ_1 and λ_2, as well as two positive real roots, say λ_3 and λ_4.[10] Consequently, the two convergent eigenvalues of the linear system in (22) can be written, respectively, as $\lambda_1 = \mathrm{Re}(\lambda_1) + \mathrm{Im}(\lambda_1)\, i$ and $\lambda_2 = \mathrm{Re}(\lambda_2) - \mathrm{Im}(\lambda_2)\, i$, with $\mathrm{Re}(\lambda_1) = \mathrm{Re}(\lambda_2) < 0$ whereas $\mathrm{Im}(\lambda_1) = \mathrm{Im}(\lambda_2)$. Obviously, $\mathrm{Re}(\lambda_k)$ ($\mathrm{Im}(\lambda_k)$) is the real (imaginary) part of λ_k, with $k = \{1, 2\}$.

In the situation under scrutiny, there is only one trajectory that satisfies the dynamic system in (14) that converges to the steady-state whereas all the others diverge. Strictly speaking, in the

[9] Along these lines, it is worth noting that higher (lower) values of m lead to an increase (reduction) of the equilibrium unionization rate.

[10] The quadratic production function of the firm and the log-linear preferences of the union rule out the possibility of limit cycles.

proposed dynamic insider-outsider model the equilibrium path is locally determinate, i.e. taking a given initial value of the insider labour force $\left(\overline{L}_I\right)$ and the transversality condition $\Psi(0) = 0$, there is a unique vector $\begin{pmatrix} \Lambda(0) & \Gamma(0) \end{pmatrix}$ in the neighbourhood of $\{\Lambda^*, \Gamma^*\}$ that generates a trajectory converging towards $\{L_I^*, \Lambda^*, \Gamma^*, \Psi^*\}$ by means of damped oscillations. Specifically, the values of $\Lambda(0)$ and $\Gamma(0)$ should be selected to satisfy the transversality conditions in (7) and (13) by placing the system in (14) exactly on the stable branch of the saddle point $\{L_I^*, \Lambda^*, \Gamma^*, \Psi^*\}$. In the remainder of the paper, the stable saddle path followed by $L_I(t)$, $\Lambda(t)$, $\Gamma(t)$, $\Psi(t)$, and, implicitly, by $w_I(t)$ and $L_E(t)$, will be taken as the perfect-foresight path of the model economy and it will be explored numerically.

3 Quantitative implications

In order to offer a quantitative assessment of the dynamic properties of the insider-outsider model developed in the previous Section, I resort now to some numerical simulations. Consequently, I provide a suitable calibration for the model economy. Thereafter, I show how the insider labour force, its wage and the number of entrants evolve over time by shaping the paths of union membership and union wage premia. Moreover, I explore the role of discounting in the determination of steady-state solutions. All the MATLAB codes used below are available from the author upon request.

3.1 Calibration

Following Shimer (2005), I map the period of the model economy into quarterly figures. Thereafter, for reasons of data availability, the theoretical setting is calibrated by taking as reference the US economy. Despite the lower unionization rate with respect to European countries, the applicability of the insider-outsider theory to the US labour market is explicitly acknowledged by Solow (1985) as well as by Lindbeck and Snower (2002).

In detail, the values of the two discount rates, i.e. ρ_F and ρ_U, are taken from Giammarioli (2003) who provides parameter values for a non-Walrasian dynamic model of the labour market developed in continuous time.[11] The separation rate is set according to the JOLT-based estimations retrieved by Shimer (2005). Thereafter, the transformation rate of entrants into insiders is set is order to replicate the union membership rate observed over the last twenty years, i.e. $m/(m + 0.10) = 12.43\%$ (cf. Hirsh and Macpherson, 2016). The implied value of the transformation rate reveals that the large majority of entrants lose their job just after one instant of employment. At the outset of the game, the consequent outflow pattern would appear to be consistent with an implausible high value of the unemployment rate. As argued by Zanetti (2007) and Cole and Rogerson (1999), however, a rationale for that high unemployment can be given by assuming that non-confirmed entrants are unconstructive searchers, i.e. individuals

[11] A similar value for the discount rate is also used in the continuous-time union model set forth by Alvarez and Shimer (2014).

that look for a job just to claim unemployment benefits whose continued employment is not profitable for the firm (cf. Coe and Snower, 1997; Guerrazzi and Ksebi, 2019).

Moreover, the slope of labour demand is calibrated by means of the value of the capital share provided by Kydland and Prescott (1982). The hypothesis underlying the calibration of α is that the production function in eq. (2) can be seen as the integration of log-linear demand schedules for insiders and outsiders. The productivity differential between insiders and outsiders is fixed according to the US union wage premium estimated by Blanchflower and Bryson (2004). The figure of the reservation wage is set according to the point estimation of the US replacement rate provided by van Vliet and Caminada (2012). The slope of marginal turnover costs is set at an intermediate level among figures provided by Oi (1962) and Barron et al. (1985). Union's preferences are calibrated according to the estimates derived by Pencavel (1985) for a large sample of Swedish firms. Taking into account the increasing path of R&D expenditure observed in the US over the last twenty years, the employment orientation implied by the retrieved value of β can be exploited to describe the rent-seeking behaviour of unions also in that country (cf. Nair-Reichert, 2000). Finally, the value of TFP is calibrated in order to replicate the chosen union wage premium, i.e. $w_E/w_I^* = \phi = 0.83$. In this way, $1/\phi$ becomes a measure of the equilibrium degree of monopoly held by the union of insiders in the wage setting process.

The whole set of parameter values and their respective description is given in Table 1.

PARAMETER	DESCRIPTION	VALUE
ρ_F	*Firm's discount rate*	0.0300
ρ_U	*Union's discount rate*	0.0300
b	*Separation rate*	0.1000
m	*Transformation rate*	0.0142
ϕ	*Productivity differential*	0.8300
α	*Labour demand slope*	0.3600
w_E	*Reservation wage*	0.5700
h	*Slope of marginal hiring costs*	0.2850
β	*Union's employment weight*	0.6490
A	*TFP*	0.8344

Table 1: Calibration

The parameter values in Table 1 lead to the following steady-state solutions: $L_I^* = 0.0303$, $\Lambda^* = 1.0525$, $\Gamma^* = 163.5066$ and $\Psi^* = -0.5112$. Those figures imply that $w_I^* = 0.6867$ and $L_E^* = 0.2132$, whereas the two convergent roots are equal respectively to $\lambda_1 = -0.1071 + 0.0395i$ and $\lambda_2 = -0.1071 - 0.0395i$. An interesting feature of the model economy disclosed by the baseline calibration is that in the stationary equilibrium the instantaneous marginal productivities of the two categories of workers are strictly positive and higher than the respective retributions. Namely, $A - \alpha L_I^* > w_I^*$ and $A\phi - \alpha L_E^* > w_E$. Such a pattern is standard in dynamic models in

which employment adjustments are subject to labour turnover costs and in the present context it is the consequence of the fact that the optimal recruiting policy of the firm – summarized by the actual value of L_E^* – has to consider the payment of the implied hiring costs (cf. Bertola, 1992).

3.2 The dynamics of L_I, L_E and w_I

As I argued above, in the neighbourhood of $\{L_I^*, \Lambda^*, \Gamma^*, \Psi^*\}$ the dynamic system in (14) is characterized by complex saddle-path dynamics. Let $\mathbf{V}(\lambda_1)$ and $\mathbf{V}(\lambda_2)$ be the eigenvectors associated, respectively, to the convergent roots λ_1 and λ_2. In our case, $\mathbf{V}(\lambda_1)$ and $\mathbf{V}(\lambda_2)$ are 4×1 arrays whose components may be real and/or complex. Thereafter, following Stemp and Herbert (2006), the analytical solution of the linearized dynamic system in (22) can be written as

$$\begin{pmatrix} L_I(t) \\ \Lambda(t) \\ \Gamma(t) \\ \Psi(t) \end{pmatrix} = \begin{pmatrix} L_I^* \\ \Lambda^* \\ \Gamma^* \\ \Psi^* \end{pmatrix} + \begin{bmatrix} V_1(\lambda_1) & V_1(\lambda_2) \\ V_2(\lambda_1) & V_2(\lambda_2) \\ V_3(\lambda_1) & V_3(\lambda_2) \\ V_4(\lambda_1) & V_4(\lambda_2) \end{bmatrix} \begin{pmatrix} \Phi(D_a, D_b, \lambda_1, t) \\ \Omega(D_a, D_b, \lambda_2, t) \end{pmatrix} \qquad D_a, D_b \in \Re \tag{25}$$

where $\Phi(D_a, D_b, \lambda_1, t)$ and $\Omega(D_a, D_b, \lambda_2, t)$ are given, respectively, by

$$\begin{pmatrix} \Phi(D_a, D_b, \lambda_1) \\ \Omega(D_a, D_b, \lambda_2) \end{pmatrix} \equiv \begin{pmatrix} (D_a + iD_b)\exp(\mathrm{Re}(\lambda_1))(\cos(\mathrm{Im}(\lambda_1)t) + i\sin(\mathrm{Im}(\lambda_1)t)) \\ (D_a - iD_b)\exp(\mathrm{Re}(\lambda_2))(\cos(\mathrm{Im}(\lambda_2)t) - i\sin(\mathrm{Im}(\lambda_2)t)) \end{pmatrix} \tag{26}$$

Setting the values of the two constants D_a and D_b consistently with the selected initial condition for $L_I(t)$ and $\Psi(t)$, eq. (25) provides the real-valued solutions $L_I(t)$, $\Lambda(t)$, $\Gamma(t)$ and $\Psi(t)$ that are necessary to derive the evolution over time of $L_E(t)$ and $w_I(t)$. The two elements in the array (26) clearly show that the analytical solution of the dynamic system in (21) has cyclical properties in the sense that $L_I(t)$, $L_E(t)$ and $w_I(t)$ converge to their respective steady-state references by means of damped oscillations whose amplitude depends on the absolute values of the imaginary parts of λ_1 and λ_2.

Fixing $D_a = 5.325$ (6.2058), $D_b = -32.3534$ (-32.1900) in (25) and exploiting the parameters values collected in Table 1, we have that $\Psi(0) = 0$ whereas $L_I(0)$ is 1% above (below) L_I^*.[12] Thereafter, the implied trajectories of L_I (left scale), L_E (left scale) and w_I (right scale) in terms of the respective deviations from steady-state values are plotted in the two panels of Figure 2, where in the panel on the left (right) the initial value of the stock of insiders starts 1% above (below) L_I^*.

[12] In order to have a trajectory for $w_I(t)$ with positive values only, D_a and D_b are fixed to meet the transversality condition in eq. (17) in a negative neighbourhood of 0.

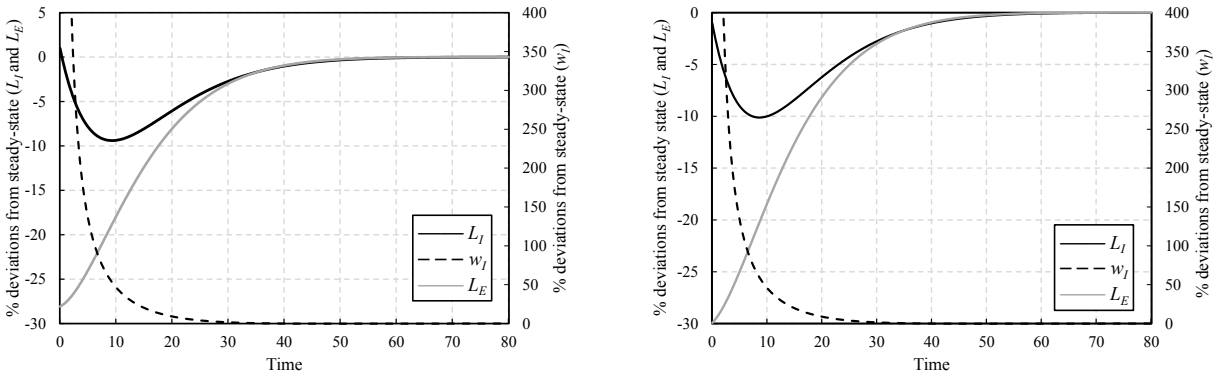

Figure 2: Saddle path adjustments of L_I, L_E and w_I

The two plots in Figure 2 shows that when the insider labour force (the black curve) starts 1% above or below its steady-state value the flow of entrants (the grey curve) jumps about 30% below its steady-state reference whereas the insider wage (the dashed curve), consistently with the theoretical findings outlined in Section 2, deeply overshoots its stationary level. The insensibility to initial conditions displayed by the model is due to the aggressive rent-seeking behaviour undertaken by the union at the beginning of the game that shapes in a very marked way the dynamics of insider wages.

These simulation results illustrated above suggest four general conclusions. First, the stock of incumbents and their wage tend to move in opposite directions during the adjustment process towards the stationary solution. This means that along the convergence path towards the steady-state the union of insiders trades off higher wages against lower membership and vice-versa (cf. Delacroix, 2006). Second, keeping the stock of incumbents as well as the flow of entrants below their stationary references, the union is able to negotiate an insider wage that remains above its steady-state value during the whole adjustment process. This finding implies a continuum decline of the union wage premium and resembles the one obtained by Zanetti (2006) in his efficient bargaining model where a reduction in the labour input results in an upward wage pressure. Third, between the two categories of workers, out-of-equilibrium adjustments tend to involve more the flow of non-unionized workers than the stock of insiders. Specifically, there is threshold – around 10% below its steady-state value – under which the stock of incumbents does not fall whereas the flow of entrants undershoot its stationary reference of about 30%. That latter finding is somehow related to the sustained turnover rate that involves entrant workers, but it is also ascribable to the optimal wage policy of the union that leads the firm to delay new hirings (cf. Kugler and Saint-Paul, 2000). As argued by Blanchard and Summers (1986), the union of insiders exerts its wage pressure to exclude disenfranchised outsiders. Moreover, corroborating findings retrieved in dynamic and static insider-outsider modes, the oscillations towards the steady-state generated by the differential game descrived above display a significant asymmetric pattern (cf. Huizinga and Schiantarelli, 1992; Begg et al. 1989).

The fact that at the beginning of the game the stock of incumbents as well as the flow of entrants tend to undershoot their steady-state values has intriguing implications for the

dynamics of the membership rate. Specifically, as shown in Figure 3, the theoretical model is fairly able to replicate the falling path of the membership rate observed in the US over the last 20 years.

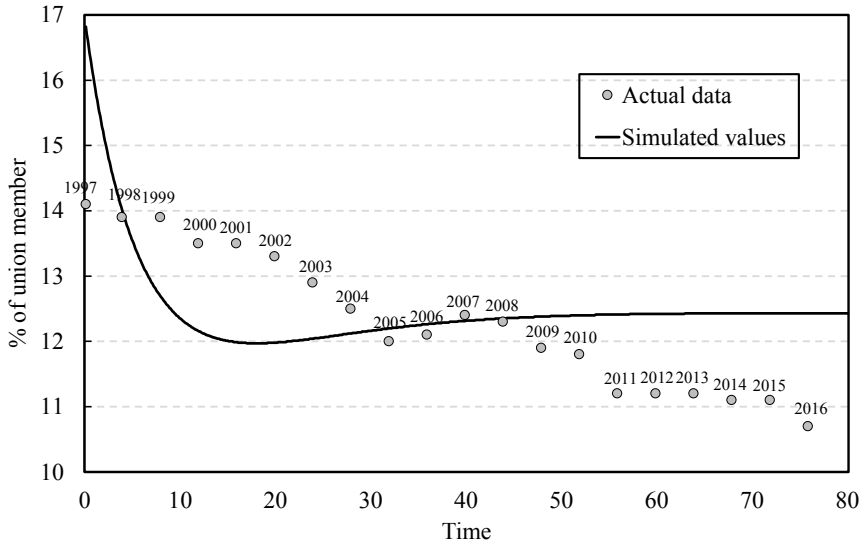

Figure 3: Membership dynamics, actual data and simulated trajectory

The plot in Figure 3 – together with the one in Figure 2 – reveals that the recent dynamics of US industrial relations, where the decline in union membership paralleled the reduction of union wage premia, can be interpreted as the outcome of the optimal behaviour of corporative unions that pushed up insider wages by restraining the expansion of output and employment (cf. Alvarez and Shimer, 2014). In fact, the insider-outsider conflict prevailing in the labour market may have rendered the incumbent labour force a form of firm-specific capital that remained subject to a severe hold up problem (cf. Oi 1962; Krusell and Rudanko, 2016). Consequently, the reduction of production induced by the rent-seeking behaviour carried out by unionized insiders can be pointed out as one of the factors that, in the long run, triggered the reduction of union membership and union wage premia (cf. Blanchflower and Bryon, 2004).

3.3 The impatience of the firm and the one of insiders

A remarkable feature of the differential game developed in Section 2 is that it allows to consider how the degree of impatience of the firm and the one of unionized insiders affects the equilibrium values of the different variables of the model economy. With regard to the literature reviewed in the introduction, this aspect may be of some interest since it is usually assumed that workers and firms discount future stream of income at the same rates (cf. Solow, 1985; Krusell and Rudanko, 2016). Through this channel, the model offers a possible rationale on how interest rates may directly affect labour market outcomes.

Taking different values for ρ_F and fixing the other parameters at the values collected in Table 1, in Table 2 I reported the corresponding figures for L_I^*, w_I^* and L_E^*.

ρ_F	L_I^*	w_I^*	L_E^*
0.01	0.0332	0.6037	0.2338
0.02	0.0316	0.6457	0.2225
0.03	0.0303	0.6867	0.2132
0.04	0.0291	0.7268	0.2053
0.05	0.0282	0.7663	0.1985

Table 2: Steady-state values of L_I^*, w_I^* and L_E^* for different values of ρ_F

Consistently with the findings outlined, respectively, by Binmore et al. (1986) and Osborne and Rubinstein (1990) within dynamic wage bargaining models, the numerical results displayed in Table 2 reveal that the higher (lower) the impatience of the single player, the lower (higher) its labour market power. Specifically, for higher (lower) values of ρ_F, the insider wage increases (decreases) while the stock of incumbents as well as the flow of entrants decrease (increase). Therefore, the lower (higher) the values of L_I^* and L_E^* and the lower (higher) the values of w_I^*, the lower (higher) the profits of the firm achieved in the stationary equilibrium. Interestingly, these findings hold no matter the employment- or the wage-orientation of the insider union, i.e. they do not depend on the actual value of β and they can also be easily framed in a more general setting where produced output is obtained not only by means of labour but even with the use of physical capital. Lower discount rates on the side of the firm are usually associated with lower interest rates that may lead entrepreneurs to substitute labour for capital. Obviously, when the labour input becomes less important in the production process because of the adoption of new automation technologies, the influence of unions is seriously undermined (cf. Dosi and Virgillito, 2019).

4 Concluding remarks

In this paper, I develop a differential insider-outsider game with continuous time and infinite horizon. Specifically, I set forth an analytically tractable dynamic model in which a union of corporative insiders is called in to choose the common trajectory of the wage paid to its members by taking into account the optimal hiring policy of a firm that, in turn, is assumed to decide the number of outsiders that has to be hired in a spot labour market characterized by the existence of quadratic hiring costs (cf. Solow, 1985; Lindbeck and Snower, 1989; Hamermesh, 1995).

Assuming that incumbents cannot be fired and commit themselves to a given path of wages, the implementation of the maximum principle reveals the possibility to find out an open-loop Stackelberg equilibrium in which the initial stock of incumbents – supplemented by a trasversality condition on the shadow value that incumbents attach to their marginal contribution to firm's profits – pins down the trajectories of the insider labour force, the one of entrants as well as the one of insider wages. The structure of the game reveals the non-trivial fact that

the union of internal workers is actually able to control the firm problem, i.e. the union sets a wage for its members that leads the firm to hire a number of entrants that is consistent with the welfare of the union itself (cf. Dockner et al. 2000; Krussel and Rudanko, 2016).

Numerical simulations obtained by calibrating the model according to selected features of the US economy show that adjustments towards the steady-state equilibrium occur through damped asymmetric oscillations that offer some insights on the simultaneous decline of union membership and union wage premia observed over the last twenty years (cf. Blanchflower and Bryson, 2004). Furthermore, consistently with dynamic models of wage bargaining, the theoretical framework developed above conveys a positive equilibrium relation between the labour market power of insiders and the rate of interest used by the firm to discount future profits (cf. Binmore et al. 1986; Osborne and Rubinstein, 1990).

From a more general perspective, the path of union membership implied by the model as well as its positive equilibrium relationship between firms' discount rates and union power may be helpful in understanding the extent of the deregulation wave that hit labour, output and financial markets in the 80s and the 90s. Compressing the profits of productive firms, the aggressive rent-seeking behaviour of unions slowed down economic growth and this naturally led to a reduction of their density and their influence for the consequent impossibility to grant to unionized workers improved opportunities in terms of wages and employment in a long-run perspective. Under this scenario, unions had serious problems in resisting to the disempowerment of labour market institutions induced by the free trade interventions promoted by public authorities throughout the different sectors of the economy. Such a traumatic tendency, however, has been somehow strengthened even by the shrink of union power implied by the reduction of discount rates triggered by the fall of interest rates that initially occurred during the Great Moderation and then exacerbated some years after by the easy-money policies carried out by central banks in the afterwards of the Great Recession. Overall, together with the widespread adoption of capital-intensive production technologies, this may explain the reason why nowadays the position of trade unions in the wage setting process and the political arena is so compromised (cf. Dosi and Virgillito, 2019).

Appendix: Controllability of the firm problem

Consider the first two differential equations of the dynamic system in (14). They read as

$$\begin{aligned} \dot{L}_I(t) &= m\tfrac{A\phi - w_E + m\Lambda(t)}{\alpha + h} - bL_I(t) \\ \dot{\Lambda}(t) &= \Lambda(t)(b + \rho_F) - A + \alpha L_I(t) - \tfrac{1-\beta}{\Psi(t)} \end{aligned} \tag{A.1}$$

Taking into account eq. (10), the system of autonomous differential equations in (A.1) can be written as the following linear time-invariant (LTI) system that describes the solution of the firm problem:

$$\begin{pmatrix} \dot{L}_I(t) \\ \dot{\Lambda}(t) \end{pmatrix} = \begin{pmatrix} \frac{m(A\phi - w_E)}{\alpha + h} \\ -A \end{pmatrix} + \begin{bmatrix} -b & \frac{m^2}{\alpha+h} \\ \alpha & b + \rho_F \end{bmatrix} \begin{pmatrix} L_I(t) \\ \Lambda(t) \end{pmatrix} + \begin{pmatrix} 0 \\ -1 \end{pmatrix} w_I(t) \tag{A.2}$$

where $w_I(t)$ is the control of the union.

The controllability matrix of the LTI system in (A.2) is given by

$$H_{2\times 2} = \begin{bmatrix} 0 & -\frac{m^2}{\alpha+h} \\ -1 & -(b+\rho_F) \end{bmatrix} \tag{A.3}$$

Since H has full rank, i.e. $rank(H) = 2$, the problem of the firm is controllable by the union.

Conflict of interest statement

The corresponding author states that there is no conflict of interest.

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